CBSE Sample Question Paper Term 1

Class – XI (Session : 2021 - 22)

SUBJECT- MATHEMATICS 041 - TEST - 03

Class 11 - Mathematics

Time Allowed: 1 hour and 30 minutes **Maximum Marks: 40**

General Instructions:

1. This question paper contains three sections - A, B and C. Each part is compulsory.
2. Section - A has 20 MCQs, attempt any 16 out of 20.
3. Section - B has 20 MCQs, attempt any 16 out of 20
4. Section - C has 10 MCQs, attempt any 8 out of 10.
5. There is no negative marking.
6. All questions carry equal marks.

Section A

Attempt any 16 questions

1. Sets A and B have 3 and 6 elements respectively. What can be the maximum number of elements in A ∪ B. **[1]**

a) 3 b) 9

c) 18 d) 6

2. If 3 f (x) + 5 $f\left(\frac{1}{x}\right)=\frac{1}{x}-3$ for all non - zero x then f(x)= **[1]**

a) $\frac{1}{14}\left(\frac{3}{x}+5x-6\right)$ b) None of these

c) $\frac{1}{14}\left(-\frac{3}{x}+5x+6\right)$ d) $\frac{1}{16}\left(-\frac{3}{x}+5x-6\right)$

3. The smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n=1$ is : **[1]**

a) none of these b) 8

c) 2 d) 4

4. The two geometric means between the numbers 1 and 64 are **[1]**

a) 4 and 16 b) 8 and 16

c) 2 and 16 d) 1 and 64

5. The ratio in which the line 3x + 4y + 2 = 0 divides the distance between the lines 3x + 4y + 5 = 0 and 3x + 4y - 5 = 0 is **[1]**

a) 2 : 3 b) 1 : 2

c) 2 : 5 d) 3 : 7

6. $\lim\limits_{x\to a}\frac{x^n-a^n}{x-a}$ is equal to: **[1]**

a) na^{n-1} b) 1

c) na^n d) na

7. Consider the first 10 positive integers. If we multiply each number by –1 and then add 1 to each number, the variance of the numbers so obtained is **[1]**

a) 3.87 b) 8.25

c) 2.87 d) 6.5

8. The locus of a point, whose abscissa and ordinate are always equal is **[1]**

a) x – y = 0 b) x + y + 1 = 0

c) x + y = 1 d) none of these

9. A relation R is defined from {2, 3, 4, 5} to {3, 6, 7, 10} by: x R y ⇔ x is relatively prime to y. Then domain of R is **[1]**

a) {2, 3, 4, 5} b) {3, 5}

c) {2, 3, 5} d) {2, 3, 4}

10. If $\alpha = \frac{z}{\bar{z}}$, then $|\alpha|$ is equal to : **[1]**

a) -1 b) 0

c) 1 d) none of these

11. The 17th term of the GP 2, $\sqrt{8}$, 4, $\sqrt{32}$... is **[1]**

a) 256 b) $256\sqrt{2}$

c) $128\sqrt{2}$ d) 512

12. A line L passes through the points (1, 1) and (2, 0) and another line M which is perpendicular to L passes through the point (1/2, 0). The area of the triangle formed by these lines with y axis is : **[1]**

a) 25/8 b) 25/16

c) none of these d) 25/4

13. $\lim\limits_{x \to 1} \frac{\sin \pi x}{x-1}$ is equal to **[1]**

a) $\frac{1}{\pi}$ b) π

c) $-\pi$ d) $-\frac{1}{\pi}$

14. Let $x_1, x_2, \ldots, x_n$ be values taken by a variable X and $y_1, y_2, \ldots, y_n$ be the values taken by a variable Y such that $y_i = ax_j + b$, i = 1, 2, ..., n Then, **[1]**

a) None of these b) Var (Y) = a^2 Var (X)

c) Var (X) = Var (X) + b d) Var (X) = a^2 Var (Y)

15. If p_1 and p_2 are the lengths of the perpendiculars from the origin upon the lines x sec θ + y cosec θ = a and x cos θ - y sin θ = a cos 2 θ respectively, then **[1]**

a) $p_1^2 + p_2^2 = a^2$ b) $4 p_1^2 + p_2^2 = a^2$

c) $p_1^2 + 4p_2^2 = a^2$ d) None of these

16. The domain of definition of the function $f(x)=\sqrt{x-1}+\sqrt{3-x}$ is [1]

a) $[1,3]$
b) $(-\infty,3)$
c) (1, 3)
d) $[1,\infty)$

17. The least positive integer n such that $\left(\frac{2i}{1+i}\right)^n$ is a positive integer, is [1]

a) 2
b) 16
c) 8
d) 4

18. If a, b, c are in A.P. and x, y, z are in G.P., then the value of $x^{b-c}\, y^{c-a}\, z^{a-b}$ is [1]

a) $x^a y^b z^c$
b) 1
c) 0
d) xyz

19. The foot of the perpendicular from (2, 3) on the line 3x + 4y – 6 = 0 is [1]

a) $\left(-\frac{14}{25},-\frac{27}{25}\right)$
b) $\left(\frac{14}{25},-\frac{27}{25}\right)$
c) $\left(\frac{14}{25},\frac{27}{25}\right)$
d) $\left(-\frac{14}{25},\frac{29}{25}\right)$

20. $\lim\limits_{x\to\pi/3}\frac{\sin\left(\frac{\pi}{3}-x\right)}{2\cos x-1}$ is equal to [1]

a) $\sqrt{3}$
b) $\frac{1}{2}$
c) $\frac{1}{\sqrt{3}}$
d) $\sqrt{5}$

Section B

Attempt any 16 questions

21. If v is the variance and σ is the standard deviation, then [1]

a) $v^2=\sigma$
b) $v=\frac{1}{\sigma}$
c) $v=\sigma^2$
d) $v=\frac{1}{\sigma^2}$

22. The lines x + (k - 1)y + 1 = 0 and 2x + k^2y - 1 = 0 are at right angles if [1]

a) k > 1
b) k = 1
c) k = - 1
d) none of these

23. If f(x) = $\frac{2^x+2^{-x}}{2}$ Then f(x + y) f (x - y) is equals to [1]

a) $\frac{1}{2}\{f(2x)-f(2y)\}$
b) 1/2 {f (2x) + f(2y)}
c) $\frac{1}{4}\{f(2x)-f(2y)\}$
d) $\frac{1}{4}\{f(2x)+f(2y)\}$

24. The complex number z such that $\left|\frac{z-i}{z+i}\right|=1$ lies on [1]

a) a circle
b) None of these
c) The x-axis
d) The line y = 1

25. GM between 0.15 and 0.0015 is [1]

a) 0.15
b) 0.015
c) 1.5
d) None of these

[1]

26. $\lim_{x \to 3} \frac{x-3}{|x-3|}$ is equal to:

a) 1

b) -1

c) 0

d) None of these

27. The mean and S.D. of 1, 2, 3, 4, 5, 6 is **[1]**

a) 3, $\frac{35}{12}$

b) 3, 3

c) $\frac{7}{2}, \sqrt{\frac{35}{12}}$

d) $\frac{7}{2}, \sqrt{3}$

28. The coordinates of the foot of perpendicular from (0, 0) upon the line x + y = 2 are **[1]**

a) (1, 1)

b) none of these

c) (-1, 2)

d) (1, 2)

29. If f(x) = $\frac{x}{x-1} = \frac{1}{y}$, then f(y) = **[1]**

a) 1 + x

b) 1 – x

c) x – 1

d) x

30. The value of a such that x^2 - 11x + a = 0 and x^2 - 14 x + 2a = 0 may have a common root is **[1]**

a) 24

b) 12

c) 0

d) 32

31. If the nth term of the GP 3, $\sqrt{3}$, 1,... is $\frac{1}{243}$ then n = ? **[1]**

a) 14

b) 13

c) 12

d) 15

32. $\lim_{n \to \infty} \frac{1-2+3-4+5-6+\ldots+(2n-1)-2n}{\sqrt{n^2+1}+\sqrt{n^2-1}}$ is equal to **[1]**

a) -1

b) $\frac{1}{2}$

c) 1

d) $-\frac{1}{2}$

33. In a group of students, mean weight of boys is 80 kg and mean weight of girls is 50kg. If the mean weight of all the students taken together is 60kg, then the ratio of the number of boys to that of the girls is **[1]**

a) 2 : 3

b) 3 : 2

c) 2 : 1

d) 1 : 2

34. The amplitude of $\frac{1}{i}$ is equal to **[1]**

a) $\frac{\pi}{2}$

b) $-\frac{\pi}{2}$

c) 0

d) π

35. If a, x, b are in GP then **[1]**

a) $x = \frac{1}{2}ab$

b) x = ab

c) x^2 = ab

d) $d = \frac{1}{2}(a+b)$

36. The number of lines that are parallel to 2x + 6y – 7 = 0 and have an intercept 10 units between the coordinate axis is : **[1]**

a) 3 b) 2

c) 4 d) 1

37. Let A = $\{x \in R : x = 0, -4 \leq x \leq 4\}$ and f : $A \to R$ be defined by f(x) = $\frac{|x|}{x}$ for $x \in A$ Then A is [1]

a) $|x : -4 \leq x \leq 0|$ b) {1}

c) $|x : 0 \leq x \leq 4|$ d) {1 ,-1 }

38. If a, b are the roots of the equation $x^2 + x + 1 = 0$, then $a^2 + b^2$ = [1]

a) 1 b) 2

c) -1 d) 3

39. How many terms of the AP -5, $\frac{-9}{2}$ -4, ... will give the sum 0? [1]

a) 21 b) 18

c) 16 d) 23

40. If $\frac{(a^{n+1}+b^{n-1})}{(a^n+b^n)}$ is the arithmetic mean between unequal numbers a and b then n = ? [1]

a) 0 b) 2

c) 4 d) 1

Section C

Attempt any 8 questions

41. The set of all prime numbers is [1]

a) an infinite set b) a singleton set

c) none of these d) a finite set

42. For all x ∈ (0, 1) [1]

a) $\log_e x > x$ b) sin x > x

c) $\log_e (1+ x) < x$ d) $e^x < 1 + x$

43. If $z = \left(\frac{1+i}{1-i}\right)$, then z^4 equals. [1]

a) 0 b) - 1

c) None of these d) 1

44. Let S_n denote the sum of the cubes of the first n natural numbers and s_n denote the sum of the first n natural numbers. Then $\sum_{r=1}^{n} \frac{S_r}{s_r}$ equals [1]

a) None of these b) $\frac{n^2+3n+2}{2}$

c) $\frac{n(n+1)(n+2)}{6}$ d) $\frac{n(n+1)}{2}$

45. The two lines of regression can never be [1]

a) intersecting b) parallel

c) perpendicular d) coincident

Question No. 46 to 50 are based on the given text. Read the text carefully and answer the questions:

In an University, out of 100 students 15 students offered Mathematics only, 12 students offered Statistics only, 8 students offered only Physics, 40 students offered Physics and Mathematics, 20 students offered Physics and Statistics, 10 students offered Mathematics and Statistics, 65 students offered Physics.

46. The number of students who did not offer any of the above three subjects is **[1]**

a) 4 b) 3

c) 5 d) 1

47. The number of students who offered mathematics and statistics but not physics is **[1]**

a) 5 b) 6

c) 7 d) 4

48. The number of students who offered statistics is **[1]**

a) 34 b) 35

c) 39 d) 31

49. The number of students who offered Mathematics is **[1]**

a) 55 b) 60

c) 65 d) 62

50. The number of students who offered all the three subjects is **[1]**

a) 2 b) 5

c) 4 d) 3

Solution

SUBJECT- MATHEMATICS 041 - TEST - 03

Class 11 - Mathematics

Section A

1. **(b)** 9

   Explanation: n(A ∪ B) = n(A) + n(B) - n(A ∩ B)
   If n(A ∩ B) = 0 then n(A ∪ B) is max.
   So max number of element in A ∪ B = 9

2. **(d)** $\frac{1}{16}\left(-\frac{3}{x}+5x-6\right)$

   Explanation: $3f(x)+5f\left(\frac{1}{x}\right)=\frac{1}{x}-3$ ... (1)
   Replacing x by 1/x;
   $3f\left(\frac{1}{x}\right)+5f(x)$= x - 3 ... (2)
   Multiply eqn 1 by 3 and eqn 2 by 5 and then subtract them
   We get,
   $-16f(x)=\frac{3}{x}-5x+6$
   $f(x)=\frac{1}{16}\left(-\frac{3}{x}+5x-6\right)$

3. **(d)** 4

   Explanation: We have $\frac{1+i}{1-i}=\frac{1+i}{1-i}\times\frac{1+i}{1+i}=\frac{(1+i)^2}{1+1}=\frac{1+2i-1}{2}=\frac{2i}{2}=i$
   Therefore $\left(\frac{1+i}{1-i}\right)^n=i^n$
   By inspection we have the smallest positive integer such that $i^n=1$ is n=4.

4. **(a)** 4 and 16

   Explanation: Let the two G.M between 1 and 64 be G_1 and G_2
   Therefore, 1, G_1, G_2 and 64 are in G.P.

   64 = 1 × r^3
   $\Rightarrow r=\sqrt[3]{64}$
   ⇒ r = 4
   ⇒ G_1 = ar = 1 × 4 = 4

   And, G_2 = ar^2 = 1 × 4^2 = 16
   Therefore, 4 and 16 are the required G.M.s

5. **(d)** 3 : 7

   Explanation: Here,it is given lines
   3x + 4y + 5 = 0 ....(i)
   3x + 4y - 5 = 0 .....(ii)
   The third line is : 3x + 4y + 2 = 0 ....(iii)
   Distance between the line (i) and (iii) = $\frac{|5-2|}{\sqrt{9+16}}=\frac{3}{5}$
   Distance between the line (i) and (iii) = $\frac{|-5-2|}{\sqrt{9+16}}=\frac{7}{5}$
   Therefore the required ratio is $\frac{3}{5}:\frac{5}{5}$ or 3 : 7.

6. **(a)** na^{n-1}

   Explanation: $\lim\limits_{x\to a}\frac{x^n-a^n}{x-a}$
   $=\lim\limits_{x\to a^+}\frac{x^n-a^n}{x-a}$ [∵ f(x) exists, $\lim\limits_{x\to a}$ f(x) = $\lim\limits_{x\to a^+}$ f(x)]
   = $\lim\limits_{h\to 0}\frac{(a+h)^n-a^n}{a+h-a}$
   = $\lim\limits_{h\to 0}a^n\frac{\left[\left(1+\frac{h}{a}\right)^n-1\right]}{h}$

$= a^n \lim\limits_{h \to 0} [1 + n \cdot \frac{h}{a} + \frac{n(n-1)}{2!} \frac{h^2}{a^2} \ldots + \ldots -1]$

$= a^n \lim\limits_{h \to 0} [\frac{n}{a} + \frac{h(h-1)}{2!} \frac{h}{a^2} + \ldots]$

$= a^n \frac{n}{a}$

$= na^{n-1}$

7. **(b)** 8.25

Explanation: First 10 positive integers are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
on multiplying each number by – 1, we get
- 1, -2, -3, -4, -5, -6, -7, -8, -9, -10
on adding 1 to each of the number, we get
0, - 1, -2, -3, -4, -5, -6, -7, -8, -9

$\therefore \sum x_i = 0 -1 -2 -3 -4 -5 -6 -7 -8 -9 = -45$

and

$\sum x_i^2 = 0^2 + (-1)^2 + (-2)^2 + (-3)^2 + (-4)^2 + \ldots + (-9)^2$

But we know $\sum n^2 = \frac{n(n+1)(2n+1)}{6}$, so the above equation on applying this formula when n = 9, we get

$\Sigma x_i^2 = \frac{9(9+1)(2(9)+1)}{6} = \frac{9 \times 10 \times 19}{6} = 285$

Now we know,

$\sigma = \sqrt{\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2}$

Substituting the corresponding values, we get

$\sigma = \sqrt{\frac{285}{10} - \left(\frac{-45}{10}\right)^2}$

$\sigma = \sqrt{28.5 - 20.25}$

$\sigma = \sqrt{28.5 - 20.25}$

$\sigma = \sqrt{8.25}$

Now for variance we will square on both sides, we get

$\sigma^2 = 8.25$

Hence the variance of the numbers so obtained is 8.25

8. **(a)** x – y = 0

Explanation: The abscissa is equal to the ordinate implies x = y
Hence, the locus is x - y = 0

9. **(a)** {2, 3, 4, 5}

Explanation: Relatively prime numbers are those numbers that have only 1 as the common factor.
So, according to this definition we get to know that (2, 3), (2, 7), (3, 7), (3, 10), (4, 3), (4, 7), (5, 3), (5, 6), (5, 7) are relatively prime.
So, R = {(2, 3), (2, 7), (3, 7), (3, 10), (4, 3), (4, 7), (5, 3), (5, 6) ,(5, 7)}.
Therefore, the Domain of R is the values of x or the first element of the ordered pair.
So, Domain = {2, 3, 4, 5}

10. **(c)** 1

Explanation: Given $\alpha = \frac{z}{\bar{z}}$

Then $|\alpha| = \left|\frac{z}{\bar{z}}\right| = \frac{|z|}{|\bar{z}|} = 1$ $[\because \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}, |z| = |\bar{z}|]$

11. **(d)** 512

Explanation: Given GP is 2, $2\sqrt{2}$, 4, $4\sqrt{2}$, ...

Here, a = 2 and $= \frac{2\sqrt{2}}{2} = \sqrt{2}$

$\therefore T_{17} = ar^{16} = 2 \times (\sqrt{2})^{16} = 2 \times 2^8 = 2^9 = 512$

Therefore, the required 17th term is 512.

12. **(b)** 25/16

Explanation: The equation of the line joining the two points (x_1, y_1) and (x_2, y_2) is

$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

The given points are (1, 1) and (2, 0)

On substituting the values we get

$\frac{y-1}{0-1} = \frac{x-1}{2-1}$

On simplifying we get,

x + y - 2 = 0

The line which is perpendicular to this line is x - y + k = 0

Since it passes through (1/2, 0)

(1/2) - 0 = k

This implies k = -1/2

Hence the equation of this line is x - y - 1/2 = 0

On solving these twolines we get the point of intersection as (5/4, 3/4)

The point which line x + y - 2 = 0 cuts the Y axis is (0, 2) and the point which the line x - y - 1/2 = 0 cuts the Y axis is (0, -1/2)

Hend e the area of the triangle = [1/2] x [5/4] x [5/4] = 25/16 squnits

13. **(c)** $-\pi$

Explanation: $\lim\limits_{x \to 1} \frac{\sin \pi x}{x-1}$

$= \lim\limits_{h \to 0} \frac{\sin \pi(1+h)}{(1+h)-1}$

$= \lim\limits_{h \to 0} \frac{\sin(\pi+\pi h)}{h}$

$= \lim\limits_{h \to 0} \frac{-\sin \pi h}{h}$

$= \lim\limits_{h \to 0} -\left(\frac{\sin \pi h}{\pi h}\right)\pi$

$= -\pi$

14. **(b)** Var (Y) = a^2 Var (X)

Explanation: We have given, $y_i = ax_i + b$

Mean (Y) = $\frac{\sum f_1}{n}$

$\bar{Y} = \frac{a\sum x_n + bn}{n}$

Mean (y) = $\frac{a\sum \bar{x}}{n} + \frac{nb}{n}$

Then, Var(Y) = $\sum \frac{\left(y_1 - \bar{Y}\right)^2}{n}$

And, Var(X) = $\sum \frac{\left(\mathrm{x}_1 - \overline{\mathrm{X}}\right)^2}{\mathrm{n}}$

Var(Y) = $\frac{\sum(aX+b-a\bar{X}-b)^2}{n}$

Var(Y) = $\frac{\sum(a-a\bar{X})^2}{n}$

Var(Y) = $a^2 \frac{\sum\left(\mathrm{x}_1 - \overline{\mathrm{X}}\right)^2}{\mathrm{n}}$

Var(Y) = a^2 Var (X)

15. **(b)** $4 p_1^2 + p_2^2 = a^2$

Explanation: $4 p_1^2 + p_2^2 = a^2$

The given lines are

x sec θ + y cosec θ = a ...(i)

x cos θ - y sin θ = a 2 cos θ ...(ii)

p_1 and p_2 are the perpendiculars from the origin upon the lines (i) and (ii), respectively , hence using the formula,

$p_1 = \left|\frac{-a}{\sqrt{\sec^2\theta + \cos ec^2\theta}}\right|$ and $p_2 = \left|\frac{-a\cos 2\theta}{\sqrt{\cos^2\theta + \sin^2\theta}}\right|$

$\Rightarrow p_1 = \left|\frac{-a\sin\theta\cos\theta}{\sqrt{\sin^2\theta+\cos^2\theta}}\right|$ and $p_2 = |-a\cos 2\theta|$

$\Rightarrow p_1 = \frac{1}{2}|-a\times 2\sin\theta\cos\theta|$ and $p_2 = |-a\cos 2\theta|$

$\Rightarrow p_1 = \frac{1}{2}\ |-a\sin 2\theta|$ and $p_2 = |-a\cos 2\theta|$

$\Rightarrow 4p_12 + p_2{}^2 = a^2 (\sin^2 2\theta + \cos^2 2\theta)$

$= a^2$

16. **(a)** $[1,3]$

Explanation: Here, $x - 1 \geq 0$ and $3 - x \geq 0$

So, $x \geq 1$ and $x \leq 3$

Therefore, $x \in [1,3]$

17. **(c)** 8

Explanation: 8

Let $z = \left(\frac{2i}{1+i}\right)$

$\Rightarrow z = \frac{2i}{1+i} \times \frac{1-i}{1-i}$

$\Rightarrow z = \frac{2i(1-i)}{1-i^2}$

$\Rightarrow z = \frac{2i(1-i)}{1+1}$ $[\because i^2 = -1]$

$\Rightarrow z = \frac{2i(1-i)}{2}$

$\Rightarrow z = i - i^2$

$\Rightarrow z = i + 1$

Now, zn = (1 + i)n

For n = 2,

$z^2 = (1 + n)^2$

$= 1 + i^2 + 2i$

$= 1 - i + 2i$

= 2i ... (i)

since this is not a positive integer

For n = 4

$z4 = (1 + i)^4$

$= [(1 + i)^2]^4$

$= (2i)^2$ [Using (i)]

$= 4i^2$

= -4 ... (ii)

This is a negative integer.

For n = 8,

$z^8 = (1 + i)^8$

$= [(1 + i)^4]^2$

$= (-4)^2$ Using (ii)]

= 16

This is a positive integer.

Thus, $z = \left(\frac{2i}{1+i}\right)^n$ is positive for n = 8.

Therefore, 8 is the least positive integer such that $\left(\frac{2i}{1+i}\right)^n$ is a positive integer

18. **(b)** 1

Explanation: a, b and c are in A.P.

$\therefore$ 2b = a + c .... (i)

And, x, y and z are in G.P.

$\therefore y^2 = xz$

Now, we can write as ,

$x^{b-c}y^{c-a}z^{a-c}$

$= x^{b+a-2b}\ y^{2ab-a-a}\ z^{a-b}$ [From (i)]

$= x^{a-b}\ y^{2(b-a)}\ z^{a-b}$

$= (xz)^{a-b}(xz)^{b-a}$ [From (ii), $y^2 = xz$]

$= (xz)^0$

= 1

19. **(c)** $\left(\frac{14}{25}, \frac{27}{25}\right)$

Explanation: The equation of the line perpendicular to the given line 3x + 4y = 6 is 4x - 3y + k = 0

Since this line passes through (2, 3)

4(2) - 3(3) + k = 0

Therefore k = 1

Therefore the line which is perpendicular to the given line is 4x - 3y + 1 = 0

On solving both the equations we get, x = $\frac{14}{25}$ and y = $\frac{27}{25}$

Hence the foot of the perpendicular is $\left(\frac{14}{25}, \frac{27}{25}\right)$

20. **(c)** $\frac{1}{\sqrt{3}}$

Explanation: $\lim\limits_{x \to \frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{3}-x\right)}{2\cos x-1}$

$= \lim\limits_{h \to 0} \frac{\sin \frac{\pi}{3}-\left(\frac{\pi}{3}-h\right)}{2\cos\left(\frac{\pi}{3}-h\right)-1}$

$= \lim\limits_{h \to 0} \frac{\sin h}{2\left[\cos \frac{\pi}{3}\cos h+\sin \frac{\pi}{3}\sin h\right]-1}$

$= \lim\limits_{h \to 0} \frac{\sin h}{2\left[\frac{1}{2}\cos h+\frac{\sqrt{3}}{2}\sin h\right]-1}$

$= \lim\limits_{h \to 0} \frac{\sin h}{\cos h+\sqrt{3}\sin h-1}$

$= \lim\limits_{h \to 0} \frac{\sin h}{-2\sin^2 \frac{h}{2}+\sqrt{3}\sin h}$

Dividing N^r and D^r by h

$= \lim\limits_{h \to 0} \frac{\frac{\sin h}{h}}{-\left(2\times\frac{h}{4}\right)\left(\frac{\sin^2 \frac{h}{2}}{\frac{h}{4}}\right)+\frac{\sqrt{3}\sin h}{h}}$

$= \frac{1}{\sqrt{3}}$

Section B

21. **(c)** v = σ^2

Explanation: If v is the variance and σ is the standard deviation, then

We know that the formula of standard variance is

$\sigma = \sqrt{\text{Variance}}$

So, variance = σ^2

22. **(c)** k = - 1

Explanation: If the lines are at right angles to each other, then the product of their slopes = -1. Slope of any line = -(coefficient of x/coeffiecient of y)

Therefore the slope of line 1 = -$\frac{1}{k-1}$

The slope of line 2 = $\frac{-2}{k^2}$

Therefore $\frac{-1}{(k-1)} \times \frac{-2}{k^2}$ = - 1

That is k^2 (k - 1) = -2

i.e; $k^3 - k^2 + 2 = 0$

On factorizing we get (k + 1)(k^2 - 2k - 2) = 0
This imlpies k + 1 is a factor, hence k = -1
Hence they are at right angles if k = -1

23. **(b)** 1/2 {f (2x) + f(2y)}

Explanation: $f(x+y)f(x-y) = \left(\frac{2^{x+y}+2^{-(x+y)}}{2}\right)\left(\frac{2^{x-y}+2^{-(x-y)}}{2}\right)$

$= \left(\frac{2^{x+y}+\frac{1}{2^{x+y}}}{2}\right)\left(\frac{2^{x-y}+\frac{1}{2^{x-y}}}{2}\right)$

$= \left(\frac{2^{2(x+y)}+1}{2 \cdot 2^{(x+y)}}\right)\left(\frac{2^{2(x-y)}+1}{2 \cdot 2^{(x-y)}}\right)$

$= \left(\frac{2^{2(x+y)}2^{2(x-y)}+2^{2(x+y)}+2^{2(x-y)}+1}{4 \cdot 2^{(x+y)}2^{(x-y)}}\right)$

$= \left(\frac{2^{4x}+2^{2(x+y)}+2^{2(x-y)}+1}{4 \cdot 2^{2x}}\right)$

$= \left(\frac{2^{2x}+2^{2y}+2^{-2y}+2^{-2x}}{4}\right)$

$= \frac{1}{2}\left(\frac{2^{2x}+2^{-2x}}{2} + \frac{2^{2y}+2^{-2y}}{2}\right)$

$= \frac{1}{2}\{f(2x) + f(2y)\}$

24. **(c)** The x-axis

Explanation: $\left|\frac{z-i}{z+i}\right| = 1 \Rightarrow \left|\frac{z-i}{z+i}\right|^2 = 1$

$\Rightarrow \left|\frac{x+iy-i}{x+iy+i}\right|^2 = 1 \Rightarrow \left|\frac{x+i(y-1)}{x+i(y+1)}\right|^2 = 1 \Rightarrow \frac{|x+i(y-1)|^2}{|x+i(y+1)|^2}$

$\Rightarrow \frac{x^2+(y-1)^2}{x^2+(y+1)^2} \Rightarrow x^2 + (y - 1)^2 = x^2 + (y + 1)^2$

$\Rightarrow (y + 1)^2 - (y - 1)^2 = 0 \Rightarrow 4y = 0 \Rightarrow y = 0$

$\Rightarrow$ z lies on the x-axis

25. **(b)** 0.015

Explanation: Therefore, GM = $\sqrt{0.15 \times 0.0015} = \sqrt{\frac{15}{100} \times \frac{15}{10000}} = \sqrt{\frac{15 \times 15}{10^6}} = \frac{15}{10^3} = \frac{15}{1000}$ = 0.015.

26. **(d)** None of these

Explanation: $\lim\limits_{x \to 3} \frac{x-3}{|x-3|}$

LHL at x = 3

$\lim\limits_{x \to 3^-} \frac{x-3}{-(x-3)}$ [$\because$ |x - 3| = -(x - 3) x < 3]

= -1

RHL at x = 3

$\lim\limits_{x \to 3^+} \frac{x-3}{x-3}$ [$\therefore$ |x - 3| = x - 3, when x > 3]

= 1

LHL $\neq$ RHL

27. **(c)** $\frac{7}{2}, \sqrt{\frac{35}{12}}$

Explanation: Mean = $\frac{1+2+3+4+5+6}{6} = \frac{21}{6} = \frac{7}{2}$

S.D = $\sqrt{\frac{n^2-1}{12}} = \sqrt{\frac{36-1}{12}} = \sqrt{\frac{35}{12}}$

28. **(a)** (1, 1)

Explanation: The equation of the line perpendicular to the given line is x - y + k = 0
Since it passes through the origin,
0 - 0 + k = 0
Therefore, k = 0
Hence the equation of the line is x - y = 0
On solving these two equations we get x = 1 and y = 1

The point of intersection of these two lines is (1, 1)
Hence the coordinates of the foot of the perpendicular is (1, 1)

29. **(b)** 1 – x

Explanation: We have, f(x) = $\frac{x}{x-1} = \frac{1}{y}$

$\therefore$ y = $\frac{x-1}{x}$

Now, f(x) = $\frac{x}{x-1}$

$\Rightarrow$ f(y) = $\frac{y}{y-1} = \frac{\frac{x-1}{z}}{\frac{x-1}{x}-1} = \frac{x-1}{x-1-x}$ = $\frac{x-1}{-1}$ = 1 - x

30. **(a)** 24

Explanation: Let α be the common roots of the equations x^2 - 11x + a = 0 and x^2 - 14x + 2a = 0.

Therefore,

α^2 - 11α + a = 0 ... (i)

α^2 - 14α + 2a = 0 ... (ii)

Solving (i) and (ii) by cross multiplication, we get,

$\frac{\alpha^2}{-22a+14a}$ = $\frac{\alpha}{a-2a}$ = $\frac{1}{-14+11}$

$\Rightarrow \alpha^2 = \frac{-22a+14a}{-14+11}, \alpha = \frac{a-2a}{-14+11}$

$\Rightarrow \alpha^2 = \frac{-8a}{-3} = \frac{8a}{3}, \alpha = \frac{-a}{-3} = \frac{a}{3}$

$\Rightarrow \left(\frac{a}{3}\right)^2$ = $\frac{8a}{3}$

$\Rightarrow$ a^2 = 24a

$\Rightarrow$ a^2 - 24a = 0

$\Rightarrow$ a(a - 24) = 0

$\Rightarrow$ a = 0 or a = 24

31. **(b)** 13

Explanation: Here, we have a = 3 and r = $\frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$.

$\therefore T_n = \frac{1}{243} \Rightarrow ar^{n-1} = \frac{1}{243}$

$\Rightarrow 3 \times \left(\frac{1}{\sqrt{3}}\right)^{n-1} = \frac{1}{243}$

$\Rightarrow \left(\frac{1}{\sqrt{3}}\right)^{n-1} = \frac{1}{729} \Rightarrow \left(\frac{1}{3}\right)^{\left(\frac{n-1}{2}\right)} = \left(\frac{1}{3}\right)^6$

$\Rightarrow \frac{n-1}{2} = 6$

$\Rightarrow$ n = 13.

32. **(d)** $-\frac{1}{2}$

Explanation: $\lim\limits_{n\to\infty}\left[\frac{1-2+3-4+5-6+\ldots(2n-1)-2n}{\sqrt{n^2+1}+\sqrt{n^2-1}}\right]$

$= \lim\limits_{n\to\infty}\left[\frac{(1+3+5+\ldots 2n-1)-(2+4+6+\ldots 2n)}{(\sqrt{n^2+1}+\sqrt{n^2-1})}\right]$

$= \lim\limits_{n\to\infty}\left[\frac{\frac{n}{2}(1+2n-1)-\frac{n}{2}(2+2n)}{(\sqrt{n^2+1}+\sqrt{n^2-1})}\right]$

$= \lim\limits_{n\to\infty}\left[\frac{n^2-n(n+1)}{(\sqrt{n^2+1}+\sqrt{n^2-1})}\right]$

$= \lim\limits_{n\to\infty}\left[\frac{-n}{(\sqrt{n^2+1}+\sqrt{n^2-1})}\right]$

Dividing the numerator and the denominator by n

$= \lim\limits_{n\to\infty}\left[\frac{-1}{\sqrt{1+\frac{1}{n^2}}+\sqrt{1-\frac{1}{n^2}}}\right]$

$= \frac{-1}{1+1}$

$= \frac{-1}{2}$

33. **(d)** 1 : 2

Explanation: Let the no. of boys be x and no. of girls be y
Sum of weights of boys = 80x
Sum of weights of girls = 50y
Sum of weights of boys and girls together = 60(x + y)
Hence, 80x + 50 y = 60x + 60y
Which gives, 20x = 10 y
So, x : y = 1 : 2

34. **(b)** $-\frac{\pi}{2}$

Explanation: $-\frac{\pi}{2}$
Let z $= \frac{1}{i}$
$\Rightarrow z = \frac{1}{i} \times \frac{i}{i}$
$\Rightarrow z = \frac{i}{i^2}$
$\Rightarrow$ z = -i
Since, z(0, -1) lies on the negative imaginary axis.
Therefore, arg (z) $= \frac{-\pi}{2}$

35. **(c)** $x^2 = ab$

Explanation: Here, a, x, b are in GP $\Rightarrow \frac{x}{a} = \frac{b}{x} \Rightarrow x^2 = ab.$

36. **(b)** 2

Explanation: The slope of the given line 2x + 6y = 7 is $\frac{-1}{3}$
Hence the line which is parallel to the above line is
y = $(\frac{-1}{3})$x + c
That is the y-intercept is (0, c) and the x-intercept is (3c, 0)
Using the distance formula
$d^2 = (0 - 3c)^2 + (3c - 0)^2$
$= 10c^2$
Since the distance is given as 10, then
$100 = 10c^2$
Therefore c = $\pm$10
Since two values are possible, two lines can be drawn.

37. **(d)** {1 ,-1 }

Explanation: When -4 < x < 0
f (x) = $-\frac{x}{x}$
= -1
When 0 < x < 4
f(x) = x/x
=1
R(f) = {-1,1}

38. **(c)** -1

Explanation: Given equation: $x^2 + x + 1 = 0$
Also, a and b are the roots of the given equation.
Sum of the roots = a + b = $\frac{-\text{Coefficientof } x}{\text{Coeficient of } x^2}$ = -$\frac{1}{1}$ = -1
Product of the roots = ab = $\frac{\text{Constant term}}{\text{Coefficient of } x^2}$ = $\frac{1}{1}$ = 1
$\therefore (a + b)^2 = a^2 + b^2 + 2ab$
$\Rightarrow (-1)^2 = a^2 + b^2 + 2 \times 1$
$\Rightarrow 1 - 2 = a^2 + b^2$
$\Rightarrow a^2 + b^2 = -1$

39. **(a)** 21

Explanation: Let S_n = 0. Then, $\frac{n}{2} \cdot [2a + (n-1)d] = 0$

$\therefore \frac{n}{2} \cdot \left[2 \times (-5) + (n-1) \times \frac{1}{2}\right] = 0 \Rightarrow n \cdot \left[\frac{-21}{2} + \frac{1}{2}n\right] = 0$

$\Rightarrow$ n = 0 or $\left\{\frac{1}{2}n - \frac{21}{2} = 0 \Rightarrow \frac{1}{2}n = \frac{21}{2} \Rightarrow n = 21\right\}$

$\therefore$ Therefore sum of 21 terms is 0.

40. **(a)** 0

Explanation: Here, it is given : $\frac{(a^{n+1}+b^{n+1})}{(a^n+b^n)} = \frac{a+b}{2}$

$\Rightarrow (a^n + b^n)(a+b) = 2\left(a^{n+1} + b^{n+1}\right)$

$\Rightarrow a^{n+1} + b^{n+1} + a^n b + b^n a = 2a^{n+1} + 2b^{n+1}$

$\Rightarrow a^{n+1} - a^n b + b^{n+1} - b^n a = 0$

$\Rightarrow a^n(a-b) - b^n(a-b) = 0$

$\Rightarrow (a-b)(a^n - b^n) = 0 \Rightarrow a^n - b^n = 0 \; [\because a - b \neq 0]$

$\Rightarrow$ a^n = b^n and $a \neq b \Rightarrow n = 0$.

Section C

41. **(a)** an infinite set

Explanation: Set A = {2, 3, 5, 7,...} so it is infinite.

42. **(c)** $\log_e$ (1+ x) < x

Explanation: Let f(x) = x - log(1 + x) in [0, x]; x $\in$ (0, 1) clearly, f is continuous on [0, x] and differentiable on (0, x).

Therefore by Lagrange's mean value theorem, there exists c $\in$ (0, x) such that,

f'(c) = $\frac{\text{f(x)} - \text{f(0)}}{\text{x} - 0}$

$\Rightarrow$ 1 - $\frac{1}{c}$ = $\frac{x-\log(1+x)-0}{x}$ $\{\because$ f'(x) = 1 - $\frac{1}{1+x}\}$

$\Rightarrow$ $\frac{x-\log(1+x)}{x}$ = 1 - $\frac{1}{1+c}$ $\Rightarrow$ $\frac{x-\log(1+x)}{x}$ > 0 [$\because$ c $\in$ (0, 1) $\Rightarrow$ 1 - $\frac{1}{1+c}$ > 0]

$\Rightarrow$ x - log (1 + x) > 0 [$\because$ x $\in$ (0, 1)]

$\Rightarrow$ log (1 + x) < x

43. **(d)** 1

Explanation: 1

Let $z = \frac{1+i}{1-i}$

$z = \frac{1+i}{1-i} \times \frac{1+i}{1+i}$

$\Rightarrow z = \frac{1+i^2+2i}{1-i^2}$

$\Rightarrow z = \frac{2i}{2}$

$\Rightarrow$ z = i

$\Rightarrow$ z^4 = i^4

Since i2 = -1, we have:

$\Rightarrow z^4 = i^2 \times i^2$

$\Rightarrow$ z^4 = 1

44. **(c)** $\frac{n(n+1)(n+2)}{6}$

Explanation: Here,it is given that

S_n = sum of the cubes of first n natural numbers

s_n = Sum of the first n natural numbers

& $\sum_{r=1}^{n} \frac{S_r}{s_r} = \frac{S_1}{s_1} + \frac{S_2}{s_2} + \cdots \frac{S_n}{s_n}$

Let T_n be the nth term of the above series

$\therefore T_n = \frac{S_n}{s_n}$ ...(i)

We know that,

Sum of cubes of first n natural numbers

$S_n=\left[\frac{n(n+1)}{2}\right]^2$

and sum of first n natural numbers

$s_n=\frac{n(n+1)}{2}$

$\therefore$ eqn (i) becomes

$\mathrm{T_n}=\frac{\left[\frac{\mathrm{n(n+1)}}{2}\right]^2}{\frac{\mathrm{n(n+1)}}{2}}$

$=\frac{n(n+1)}{2}$

$=\frac{n^2+n}{2}$

Now, sum of the given series

$\sum \mathrm{T_n}=\frac{1}{2}\sum\left[\mathrm{n}^2+\mathrm{n}\right]$

$=\frac{1}{2}\sum n^2+\sum n$

$=\frac{1}{2}\left[\frac{\mathrm{n(n+1)(2n+1)}}{6}+\frac{\mathrm{n(n+1)}}{2}\right]$

$=\frac{1}{2}\times\frac{\mathrm{n(n+1)}}{2}\left[\frac{\mathrm{2n+1}}{3}+1\right]$

$=\frac{\mathrm{n(n+1)}}{4}\left[\frac{\mathrm{2n+1+3}}{3}\right]$

$=\frac{n(n+1)}{4}\left[\frac{2n+4}{3}\right]$

$=\frac{n(n+1)\times 2\times(n+2)}{4\times 3}$

$=\frac{\mathrm{n(n+1)(n+2)}}{6}$

Therefore,the correct options is $\frac{n(n+1)(n+2)}{6}$.

45. **(b)** parallel

Explanation: because $-1\leq\rho\leq 1$

So, tan θ = $\frac{1-\rho^2}{\rho}\cdot\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$

$\theta=\pi,\frac{\pi}{2},0^\circ$

So lines can't be parallel.

46. **(d)** 1

Explanation: 1

47. **(c)** 7

Explanation: 7

48. **(c)** 39

Explanation: 39

49. **(d)** 62

Explanation: 62

50. **(d)** 3

Explanation: 3